

Finite Density QCD in the Chiral Limit

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We present the first results of an exact simulation of full QCD at finite density in the chiral limit. We have used a MFA (Microcanonical Fermionic Average) inspired approach for the reconstruction of the Grand Canonical Partition Function of the theory; using the fugacity expansion of the fermionic determinant we are able to move continuously in the $(\beta - \mu)$ plane with $m = 0$.

Introduction

The finite density formulation of QCD has always been one of the most difficult problems for the lattice community. In fact the only consistent definition of the discrete partition function

$$\mathcal{Z} = \int \mathcal{D}U \det \Delta(U; m, \mu) e^{-S_g(U)} \quad (1)$$

goes through a complex fermion determinant $\det \Delta$. This means that the fermion determinant is not any more a good probability weight and the consequence is the breakdown of almost all the standard simulation algorithms. General methods for simulating systems with a complex action are tremendously time consuming and are presently inadequate to perform simulations on reasonable lattices with nowadays computing resources [1]. The quenched approximation appears to suffer from strong unphysical effects [2] as signalled from the value obtained for the critical density $\mu_c \simeq \frac{1}{2}m_\pi$ instead of $\mu_c \simeq \frac{1}{3}m_B$ as expected. The most promising approach is proposed by the Glasgow group [3] that uses the grand-canonical formulation (GCPF) and generates the gauge field configurations with the real $\mu = 0$ fermion determinant. The results now available are however still unclear: the onset density μ_o is related to m_π and a rather weak transi-

tion signal is found in the expected range [4]. To our opinion it is crucial at this point to rely on a different simulation procedure in order to check the results and, possibly, to perform zero quark mass calculations to clearly separate the m_π and m_B masses on the available lattice extensions.

The method

The idea is to consider $\det \Delta$ as an observable, avoiding the problem of dealing with a complex quantity in the generation of configurations. This can be done in a (in principle) exact way by means of the *MFA* algorithm [5] where the mean value of the determinant at fixed pure gauge energy is used to reconstruct an effective fermionic action as a function of the pure gauge energy only. The method allows free mobility in the $\beta - m$ plane, including the $m = 0$ case, and up to now this method has been successfully used in several models. We used the GCPF to write the fermionic determinant as a polynomial in the fugacity $z = e^\mu$:

$$\begin{aligned} P(U; m) &= \begin{pmatrix} -GT & T \\ -T & 0 \end{pmatrix} \\ \det \Delta(U; m, \mu) &= z^{3V} \det (P(U; m) - z^{-1}) \\ &= \sum_{n=-3L_s^3}^{3L_s^3} c_n z^{nL_t} \end{aligned} \quad (2)$$

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where G contains the spatial links and the mass term, T contains the forward temporal links and V is the lattice volume. Once fixed m , a complete diagonalization of the P matrix allows to reconstruct $\det \Delta$ for all the values of μ .

Due to the $Z(L_t)$ symmetry of the eigenvalues of P it is possible to write P^{L_t} in a block matrix form and we only need to diagonalize a $(6L_s^3 \times 6L_s^3)$ matrix. The configurations are generated at fixed β using only the pure gauge action and a standard Cabibbo-Marinari pseudo heat-bath. Tuning β appropriately we can easily generate configurations in a $O(1/V)$ interval around the desired pure gauge energy so to have well decorrelated, fixed pure gauge energy configurations. Then we can calculate $\langle \det \Delta(U; m, \mu) \rangle_E$ for a set of energies and reconstruct the fermionic effective action using this quantity and the pure gauge density of states obtained with standard (pure gauge) canonical simulations. Interpolating this quantity and performing one dimensional integrals we can calculate the partition function and its derivatives in a range of β and for all the values of μ at negligible computer cost. The chiral limit is straightforward since it only accounts in diagonalizing $P(U; m = 0)$.

The simulations

We have performed simulations on 4^4 and 6^4 lattices with four flavours of Kogut-Susskind fermions, antiperiodic boundary conditions in time and periodic ones in the other directions. In order to have real defined quantities for the mean value of the observables we need $\langle \det \Delta(U; m, \mu) \rangle_E \in \mathbb{R}$.

This comes out not to be the case and in fact what we get in the confined region with some hundreds of configurations is a wildly fluctuating phase for the effective action at intermediate μ . We can imagine several different definitions to overcome the problem and get a real quantity. One possibility is to use the mean value of the modulus of the determinant

$$\langle |\det \Delta(U; m, \mu)| \rangle_E = \langle \det \Delta(U; m, \mu) \rangle_{\parallel E}. \quad (3)$$

This definition can lead to wrong results if the mean value of the cosine of the determinant

phase, weighted with the modulus of the determinant itself, is a quantity that goes to zero exponentially with the volume. This is the case in one dimensional QED [6] but, due to the discrete nature of the center of the gauge group, it is not true in one dimensional $SU(N_C)$ models. For the $N_C = 3$ case the determinant is written:

$$\det \Delta(U; \mu) = \prod_{j=1,2,3} (2 - e^{i\theta_j + V\mu} - e^{-i\theta_j - V\mu}) \quad (4)$$

where θ_j are the phases of the eigenvalues of a $SU(3)$ matrix and $\theta_1 + \theta_2 + \theta_3 = \pi$ for antiperiodic boundary conditions. In this model the phase of the determinant is a finite volume effect and $\det \Delta$ becomes real for each configuration as V goes to infinity.

In the present work we used the quantity in (3) to define the partition function and calculate plaquette, chiral condensate, number density.

The chiral condensate has been obtained diagonalizing the same set of configurations for several different masses and substituting the derivative respect to the mass with finite differences:

$$\langle \bar{\psi} \psi \rangle_{\parallel} \simeq \frac{\ln Z_{\parallel}(m_1, \mu) - \ln Z_{\parallel}(m_2, \mu)}{(m_2 - m_1)VN}. \quad (5)$$

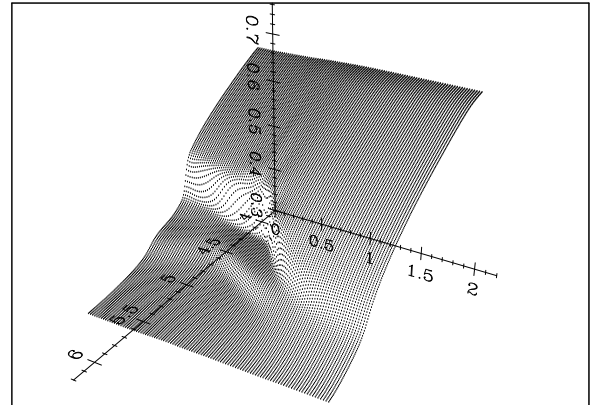


Figure 1. $\langle E \rangle_{\parallel}$ in a 4^4 lattice in the $\beta - \mu$ plane ($m = 0$, $\mu \in [0, 2]$).

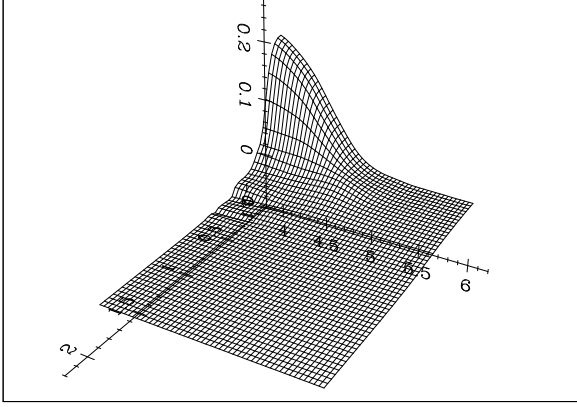


Figure 2. $\langle \bar{\psi}\psi \rangle_{\parallel}$ in a 4^4 lattice ($\mu \in [0, 2]$) with $m_1 = 0$ and $m_2 = 0.025$ (see (5)).

Conclusion

In fig.1 we present the plot of the plaquette. The two edges $\mu = 0$ and $\mu = 2$ correctly reproduce the known results of the unquenched and quenched zero density theory. In fact, for all the gluonic observables, as $\mu \rightarrow \pm\infty$ the fermionic determinant becomes independent of E , factorizes out and we recovery the pure gauge results.

Fig.2 shows the chiral condensate for the 4^4 lattice. Moving away from the $\beta = 4, \mu = 0$ point it drops to zero along both axes signalling the finite density and finite temperature chiral restoration. In order to study the possible relation between the finite density critical point to m_{π} , in Fig. 3 we show the number density at intermediate coupling for different lattices and, for the smaller one, for different masses. The value μ_o , where $n(\mu)$ starts to be different from zero, vanishes in the chiral limit; increasing the volume does not modify this scenario. This suggests that the onset density is correlated with the pion mass instead of the baryon mass, in contrast with the result for the chiral restoration transition which appears to occur at a non vanishing μ in the chiral limit.

At first glance the modified *MFA* algorithm seems to produce consistent results and shares the problem of early onset with the Glasgow method.

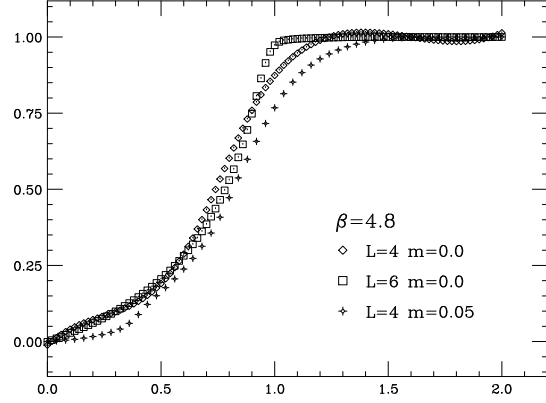


Figure 3. $\langle n(\mu) \rangle_{\parallel}$ for 4^4 and 6^4 lattices at $\beta = 4.8$ and two masses. Errors smaller than symbols.

One of the potentialities of the *MFA* method is the possibility to use the same data set to perform analysis with different definitions for the effective action (e.g. $|\langle \det \Delta(U; m, \mu) \rangle_E|$ or the modulus of the coefficients c_n) without extra computer cost. To our opinion an important step would be to complete this analysis using different definitions for the fermionic effective action and compare the results, to clarify the role of the phase of the determinant in the thermodynamic limit of the model.

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